2017

MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) Write the nth derivative of $\sin^3 x$.

(b) If $f(x, y) = 3x^2y + 2xy^2$, find $f_x(1, 2)$.

(c) State Euler's theorem on homogeneous function of degree n for two variables.

(d) Write the subtangent of the curve $y^2 = 4ax$.

(e) Define asymptotes.

(f) Write the value of $\int_{-a}^{a} x^3 \sqrt{a^2 - x^2}\, dx$.

(g) Define point of inflexion.

(h) For a pedal curve $p = r \sin \phi$, write the formula for radius of curvature.

(i) Write down the reduction formula for

$$\int \tan^n x\,dx$$

(j) What is a cusp?

2. Answer the following questions : 2×5=10

(a) Find nth derivative of $\dfrac{1}{a^2 - x^2}$.

(b) If $u = x^2 \tan^{-1}\dfrac{y}{x} - y^2 \tan^{-1}\dfrac{x}{y}$, find

$$\frac{\partial^2 u}{\partial x \partial y}$$

(c) The tangent of the curve $y^2 = 4a\left\{x + \sin\dfrac{x}{a}\right\}$ at (x_1, y_1) is parallel to x-axis. Show that

$$\cos(x_1 / a) = -1$$

(d) Evaluate $\int_0^{\pi} x \sin x \cos^2 x\,dx$.

(e) Find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.

3. Answer the following questions :

(a) (i) If $u = e^{xyz}$, show that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2)e^{xyz}$$ 3

(ii) Find the pedal equation of the curve

$$x^2 + y^2 = 2ax$$ 2

(b) Derive a reduction formula for $\int \cos^n x\,dx$. 5

4. Answer either (a) or (b) :

(a) (i) Tangents are drawn from the origin to the curve $y = \sin x$. Prove that their points of contact lie on

$$x^2 y^2 = x^2 - y^2$$ 5

(ii) Evaluate $\displaystyle\int \frac{dx}{(1+x)\sqrt{1 + 2x - x^2}}$. 5

(b) (i) Evaluate $\displaystyle\int \frac{dx}{3 + 5\cos x}$. 5

(ii) Evaluate $\displaystyle\int \sqrt{\frac{x-3}{x-4}}\,dx$. 5

5. Answer the following questions :

(a) If $y = [x + \sqrt{1 + x^2}]^m$, find the nth derivative of y for $x = 0$. 5

(b) Find the perimeter of the circle

$$x^2 + y^2 = a^2$$ 5

6. Answer either (a) or (b) :

(a) (i) If $u = x\phi(y/x) + \psi(y/x)$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 y}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$$ 5

(ii) Find the volume of the solid generated by the revolution of the curve $(a-x)y^2 = a^2 x$ about its asymptote. 5

(b) (i) Find the asymptotes of the curve

$$x^4 - x^2y^2 + x^2 + y^2 - a^2 = 0$$ 5

(ii) Trace the curve $y = x^3$. 5

7. Answer the following questions :

(a) Show that points of inflexion of the curve $y^2 = (x-a)^2(x-b)$ lie on the line $3x + a = 4b$. 5

(b) Find the surface area of the solid generated by revolving the cardioid $r = a(1-\cos\theta)$ about the initial line. 5

8. Answer either (a) or (b) :

(a) Derive a reduction formula for

$$\int \sin^m x \sin nx\, dx$$

Hence evaluate

$$\int_0^{\pi} \sin^m x \sin nx\, dx$$ 7+3=10

(b) What are the double points? Examine the nature of double points of the curve

$$2(x^3 + y^3) - 3(3x^2 + y^2) + 12x = 4$$ 2+8=10

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