# 2019

## STATISTICS

## ( Major )

Paper : 2.2

**( Mathematical Method—I )**

*Full Marks* : 60

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

**1.** Answer the following as directed : 1×7=7

*(a)* Define gamma integral.

*(b)* In a finite interval which encloses no point of infinite discontinuity, the integrand is _____ and _____.

( Fill in the blanks )

*(c)* When does an integral

$$\int_0^1 x^{m-1}(1-x)^{n-1}dx$$

exist?

*(d)* Define upper and lower Riemann sums of a function corresponding to the partition $P$.

*(e)* Every uniformly convergent sequence is pointwise convergent and the uniform limit function is same as the pointwise limit function.

( Write True or False )

*(f)* State the necessary condition for $f(x)$ to have an extreme value at the point $C$.

*(g)* If $f(xy) = 2x^2 - xy + 2y^2$, then find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point (1, 2).

**2.** Answer the following : 2×4=8

*(a)* State the geometrical (physical) interpretation of Cauchy's mean value theorem.

*(b)* If a function $f$ is twice derivable on $[a, a+h]$, then show that

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2} f''(a + \theta h), \quad 0 < \theta < 1$$





*(c)* Examine the convergence of

$$\int_0^1 \frac{dx}{\sqrt{1-x}}$$

*(d)* Show that

$$\int_a^b (x-a)^{l-1}(b-x)^{m-1} dx = (b-a)^{l+m-1} \beta(l, m)$$

**3.** Answer any *three* questions : 5×3=15

*(a)* If a function $f$ defined on $[a, b]$ is *(i)* continuous on $[a, b]$, *(ii)* derivable on $]a, b[$ and *(iii)* $f(a) = f(b)$, then prove that there exists at least one real number $\xi$ between $a$ and $b$, $(a < \xi < b)$ such that $f'(\xi) = 0$.

*(b)* Show that the sequence $\{f_n\}$, where $f_n(x) = x^n$, is uniformly convergent on $[0, k]$ and only pointwise convergent on $[0, 1]$.

*(c)* If a function $f$ defined on $[a, a+h]$ is *(i)* continuous on $[a, a+h]$ and *(ii)* derivable on $]a, a+h[$, then prove that there exists at least one number $\theta \in ]0, 1[$ such that

$$f(a+h) = f(a) + hf'(a + \theta h), \quad \theta \in ]0, 1[$$

(d) (i) Show that

$$\int_a^b \frac{dx}{(x-a)^P}$$

converges, if $P<1$ and diverges, if $P \geq 1$. 3

(ii) Find the value of

$$\int_0^{\pi} \cos^4 \theta \, d\theta$$ 2

(e) Show that $x^5 - 5x^4 + 5x^3 - 1$ has a maximum at $x = 1$ and minimum at $x = 3$ and neither at $x = 0$.

4. Answer any *three* questions : 10×3=30

(a) (i) State and prove Cauchy's criterion for uniform convergence. 6

(ii) Examine the validity of the hypothesis and the conclusion of Lagrange's mean value theorem for the function

$$f(x) = x(x-1)(x-2) \text{ on } \left[0, \frac{1}{2}\right]$$ 4





(b) (i) Show that

$$\frac{\Gamma(z)\Gamma(a+1)}{\Gamma(z+a)} = \sum_{n=0}^{\infty} (-1)^n \frac{a(a-1)(a-2)\ldots(a-n)}{n!} \frac{1}{z+n}$$ 6

(ii) Show that

$$\int_0^{\infty} \frac{x^{m-1}}{(a+bx)^{m+n}} dx = \frac{1}{a^n b^m} \beta(m, n)$$ 4

(c) (i) If $\alpha, \beta, \gamma$ are the roots of the equation in $t$, such that

$$\frac{u}{a+t} + \frac{v}{b+t} + \frac{w}{c+t} = 1$$

then prove that

$$\frac{\partial(u, v, w)}{\partial(\alpha, \beta, \gamma)} = \frac{(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)}{(b-c)(c-a)(a-b)}$$ 6

(ii) Show that $x^2$ is integrable on any interval $[0, k]$. 4

(d) (i) Prove that the function $f(xy) = |xy|^{\frac{1}{2}}$ is not differentiable at the point (0, 0), but that $f_x$ and $f_y$ both exist at the origin and have the value zero. 5

(ii) A function $f$ is bounded and integrable on $[a, b]$ and there exists a function $\phi(x)$ such that $\phi'(x) = f(x)$ on $[a, b]$, then prove that

$$\int_a^b f(x)\, dx = \phi(b) - \phi(a)$$ 5

(e) (i) Write a note on Lagrange's method of undetermined multiplier. 6

(ii) If $z = e^{xy^2}$, $x = t\cos t$, $y = t\sin t$, find $\frac{\partial z}{\partial t}$ at $t = \frac{\pi}{2}$. 4

(f) State and prove Tylor's theorem for two variables. 10

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