2019

STATISTICS

(Major)

Paper : 3·2

(Distribution—I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) If X is a Bernoulli random variable, then find moment-generating function.

(b) All cumulants of the Poisson distribution are equal.

(State True or False)

(c) State the relationship between mean and variance of geometric distribution.

(d) Under what conditions, beta distribution tends to uniform distribution?

(e) The ratio of two independent gamma variates is a beta variate of 2nd kind.

(State True or False)

(f) If $X \sim \exp(\theta)$, then for what value of θ the mean and variance are equal?

(g) State the relationship between hypergeometric distribution and binomial distribution.

2. Answer the following questions : 2×4=8

(a) Obtain the moment-generating function of negative binomial distribution and hence find the mean of this distribution.

(b) Obtain the probability-generating function of geometric distribution.

(c) Obtain the characteristic function of the standard normal variate.

(d) Let $X_1, X_2, \cdots, X_n$ be i.i.d. one-parameter exponential variates with mean $\frac{1}{\lambda}$ of each X_i, $i = 1, 2, \ldots n$. Then show that the sum $S = \sum_{i=1}^{n} X_i$ follows two-parameter gamma variate with parameters λ and n.

3. Answer any *three* of the following : 5×3=15

(a) If X and Y are independent Poisson variates, then show that the conditional distribution of X given $X+Y$ is binomial.

(b) Let X be a non-negative integer valued random variable satisfying $P(X > i + j | X > i) = P(X \geq j)$ for any two positive integers i and j. Then show that X must have a geometric distribution.

(c) Show that for the normal curve maximum probability occurs at the mean of the distribution.

(d) When is a variable said to follow exponential distribution? What are the properties of exponential distribution?

(e) Give the outline of lognormal distribution and give its uses.

4. Answer any *three* of the following : 10×3=30

(a) If X is a Poisson variate with parameter λ, then prove that

$$\mu_{r+1} = \lambda\left(r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right)$$

where μ_r is the rth moment about mean λ. Hence obtain the skewness and kurtosis of Poisson distribution. 10

(b) (i) Elucidate geometric distribution and give its properties. 5

(ii) Show that any linear combination of n independent normal variates is also a normal variate. 5

(c) (i) For a beta (m, n) distribution

$$f(x) = \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)} x^{m-1}(1-x)^{n-1}; 0 < x < 1$$

Verify that the harmonic mean is less than the arithmetic mean. 6

(ii) Write down the p.d.f. of the three-parameter Weibull distribution. Assuming the location parameter to be zero, derive mean and variance. 4

(d) (i) Obtain the characteristic function of standard Cauchy distribution. 5

(ii) Show that if the random variables X and Y are i.i.d. binomial variables with parameters (n, p) and (m, p) respectively, then the conditional distribution of $\frac{X}{X+Y}$ is hypergeometric. 5

(e) (i) State the applications of gamma distribution. 3

(ii) Find the mean and variance of hypergeometric distribution. 7

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