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3 (Sem–4/CBCS) STA HC 2

2023

STATISTICS

(Honours Core)

Paper : STA–HC–4026

(Linear Models)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1 \times 7 = 7$

   (a) In regression analysis, the variable that is being predicted is

   (i) the independent variable

   (ii) the dependent variable

   (iii) usually denoted by x

   (iv) usually denoted by r

   (Choose the correct option)

Contd.

(b) The coefficient of determination is

(i) equal to zero

(ii) the ratio of explained and total variation

(iii) usually less than zero

(iv) 100% of $\left(1-r^2\right)$

(Choose the correct option)

(c) In least square estimation, which of the following is not a required assumption about the error term ?

(i) The expected value of the error term is one

(ii) The variance of the error term is the same for all values of x

(iii) The values of the error term are independent

(iv) The error term is normally distributed

(Choose the correct option)

(d) If the regression equation is equal to $Y = 23.6 - 54.2X$, then 23.6 is the _____ while -54.2 is the ______ of the regression line.

(i) slope, intercept

(ii) slope, regression coefficient

(iii) intercept, slope

(iv) radius, intercept

(Choose the correct option)

(e) Analysis of variance is a statistical method of comparing the _______ of several populations.

(i) standard deviations

(ii) variances

(iii) means

(iv) None of the above

(Choose the correct option)

(f) The sum of squares due to _______ measures the variability of the observed values around their respective treatment means

(i) treatment

(ii) error

(iii) interaction

(iv) total

(Choose the correct option)

(g) All OLS estimators are linear estimators. *(Write True **or** False)*

2. Answer the following questions briefly : 2×4=8

(a) State some applications of the analysis of variance.

(b) What do you understand by components of variation ?

(c) Define estimability of linear parametric functions.

(d) Define R^2 in the context of a linear model.

3. Answer ***any three*** of the following questions : 5×3=15

(a) What is a linear model ? Discuss different types of linear models.

(b) A sample of 20 observations on X and Y gave the following data :

$$\sum Y = 21.9 \qquad \sum(Y-\bar{Y})^2 = 86.9$$
$$\sum X = 186.2 \qquad \sum(X-\bar{X}) = 215.4$$
$$\sum(X-\bar{X})(Y-\bar{Y}) = 106.4$$

Estimate the regression equation of Y on X and X on Y.

(c) Consider the one-way AOV model $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$, for $i = 1,2$ and $j = 1,2,3$ Examine if μ, α_1, α_2 are estimable without any constraints.

(d) In what respects do AOV, regression analysis and AOCOV differ ? Discuss briefly.

(e) Write a note on the technique of hypothesis testing in case of simple regression models.

4. Answer **either** *(a)* **or** *(b)* : 10

(a) State and prove the Gauss-Markov theorem.

(b) What is analysis of variance (AOV) ? What are the basic assumptions associated with it ? What are the remedies, if the assumptions are violated ?

5. Answer **either** *(a)* **or** *(b)* : 10

(a) Define a linear regression model. Write the basic assumptions of the linear model. Estimate the parameters of the model.

(b) Give linear model (fixed effect) for two-way classification (one observation per cell) and state its assumptions. Derive the analysis of variance of two-way classification through the method of least squares.

6. Answer **either** *(a)* **or** *(b)* : 10

(a) Using the following data

Y :	65	57	57	54	66
X :	26	13	16	−7	27

estimate the regression line $Y = \alpha + \beta X$, test the hypothesis that $\beta = 0$ against the alternative $\beta < 0$ at 5% level of significance, also construct 95% confidence interval for β.

(Given $t_{0.05,3} = 2.353$)

(b) Derive the 'analysis of covariance' for a one-way layout (with one consistent variable only).