2023

STATISTICS

(Honours Core)

Paper : STA–HC–4016

(Statistical Inference)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×7=7

   (a) Sample median is ______ estimator for the mean of normal population. *(Fill in the blank)*

   (b) Unbiased estimators are necessarily consistent. *(State True or False)*

(c) Area of critical region depends on

(i) number of observations

(ii) value of the statistic

(iii) size of type I error

(iv) size of type II error

(Choose the correct option)

(d) For a certain test if $\alpha = 0.05$, $\beta = 0.10$, then the power of the test is

(i) 0.95

(ii) 0.90

(iii) 0.05

(iv) 0.10

(Choose the correct option)

(e) Sample moments are ______ estimators of the corresponding population moments. *(Fill in the blank)*

(f) Suppose we put forward an interval which we expect to include the trees parameter value, then the process is called ______ estimation.

(Fill in the blank)

(g) The N-P lemma proceeds the best critical region for testing ______ hypothesis against ______ alternative hypothesis. *(Fill in the blanks)*

2. Answer the following questions : 2×4=8

(a) If $x_1, x_2, ...x_n$ is a random sample from a normal population $N(\mu,1)$, then show that $T = \sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $\mu^2 + 1$.

(b) Find the maximum likelihood estimator of θ for the following probability distribution :

$f(x,\theta) = \theta e^{-\theta x}$, $x > 0$, $\theta > 0$

(c) State the Neyman-Pearson lemma.

(d) Give example of a maximum likelihood estimator which is not unbiased.

3. Answer ***any three*** questions from the following : 5×3=15

(a) Obtain the M.L.E. of α and β for the rectangular distribution

$$f(x:\alpha,\beta)=\begin{cases}\frac{1}{\beta-\alpha}, & \alpha<x<p\\ u & , \text{elsewhere}\end{cases}$$

(b) Show that, if a sufficient estimator exists, it is a function of the M.L.E.

(c) What is meant by statistical hypothesis? Explain the concept of type I and type II error with example. What is the power of a test ?

(d) Let X have the p.d.f. of the form

$$f(x,\theta)=\theta\, x^{\theta-1},\ 0<x<1$$
$$=0 \quad , \text{elsewhere}$$

Find the most powerful test to test the simple hypothesis

$$H_0:\theta=1$$

against the alternative hypothesis

$$H_1:\theta=2$$

by means of a single observation X. What would be the size of type I and type II error, if you choose the interval

(i) $x\geq 0.05$

(ii) $x\geq 1.5$

as critical region ?

(e) Let $x_1, x_2, \ldots\ldots x_n$ be a random sample from a distribution with p.d.f.

$$f(x,\theta)=e^{-(x-\theta)}\ ,\theta<x<\infty$$
$$-\infty<\theta<\infty$$

Obtain a sufficient statistic for θ.

4. Answer ***any three*** questions from the following : 10×3=30

(a) What do you mean by MP and UMP tests ? Show that the most powerful test is necessarily unbiased.

(b) State the Cramer-Rao inequality. What are the conditions for equality sign in C-R inequality ? Show that,

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

in random sampling from

$$f(x,\theta) = \begin{cases} \frac{1}{\theta}\exp\left(-x/\theta\right), & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$$

where, $0 < \theta < \infty$ is an MVB estimator of θ and has variance $\frac{\theta^2}{n}$.

(c) Define consistent estimator. State and prove the sufficient condition for consistency of an estimator.

(d) Show that with the help of example,

(i) an MLE is not unique;

(ii) an MLE may not exist.

(e) What is likelihood ratio test ? Show that likelihood ratio test for testing the variances of two normal population is the usual F-test.

(f) *(i)* Describe the method of moments for estimating parameter.

(ii) Show that in sampling from Cauchy population,

$$f(x,\theta) = \frac{1}{\pi\left[1 + (x-\theta)^2\right]}, \quad -\infty < x < \infty, \quad \theta > 0$$

is not sample mean, but sample median is a consistent estimator of θ.

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