3 (Sem–4/CBCS) MAT HC 2

2023

MATHEMATICS

(Honours Core)

Paper : MAT–HC–4026

(Numerical Methods)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×7=7

   (a) What is the order of convergence of Regula-Falsi method ?

   (i) 2·312

   (ii) 2·231

   (iii) 1·618

   (iv) 1·321

   (Choose the correct option)

(b) Find $\Delta^{n+1} x^n = ?$

(c) Write down Newton's forward interpolation formula.

(d) The Newton-Raphson method is also called as

(i) tangent method

(ii) secant method

(iii) chord method

(iv) diameter method

(Choose the correct option)

(e) In the general Quadrature formula Simpson's one third rule is obtained by putting

(i) $n = 1$

(ii) $n = 2$

(iii) $n = 3$

(iv) $n = 4$

(Choose the correct option)

(f) The value of $\int_0^{\pi/4} \frac{dx}{1+x^2}$ is

(i) 0

(ii) 1

(iii) 2

(iv) None of the above

(Choose the correct option)

(g) Where is Euler's method used ?

2. Answer the following questions : 2×4=8

(a) Define rate of convergence and order of convergence of a sequence.

(b) Evaluate : $\frac{\Delta^2}{E} x^3$

(c) Construct a divided difference table from the following data :

x	−1	1	2	3
y	−21	15	12	3

(d) Why is Lagrange's formula considered to be of more general nature than Newton's formula ?

3. Answer ***any three*** questions : 5×3=15

(a) What do you mean by algorithm ? Use the statistics algorithm to compute the mean and standard deviation of the following data : 1+4=5

1 , 3 , 5 , 7 , 9

(b) Find a root of the equation

$$x^3 - 4x - 9 = 0$$

using the bisection method correct up to 3 decimal places.

(c) Show that

(i) $\delta \equiv \nabla (I - \nabla)^{-1/2}$

(ii) $E\Delta \equiv \Delta E$ 3+2=5

(d) Find the rate of convergence of Newton-Raphson method.

(e) Using Lagrange's interpolation formula for unequal interval, find the values of $f(2)$ and $f(15)$ from the following data :

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

4. Answer the following questions : 10×3=30

(a) Determine the root of

$xe^x - 2 = 0$ by the method of false position. Perform *five* iterations.

OR

Form an LU decomposition of the following matrix :

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & -2 \end{pmatrix}$$

(b) Let $x_0, x_1, \ldots x_n$ be $(n+1)$ distinct points on $[a, b]$. If f is continuous on $[a, b]$ and has n continuous derivatives on (a, b), then prove that there exist some $\xi \in (a, b)$ such that

$$f[x_0, x_1, \ldots, x_n] = \frac{f^n(\xi)}{\xi!}$$

where $f^n(x) = \frac{d^n f(x)}{dx^n}$.

Find the interpolating polynomial from the data given below using divided differences :

x	:	-2	0	2
$f(x)$	:	4	2	8

5+5=10

OR

Derive the formula for finding first and second order derivatives using Newton's forward difference formula.

Given that

X :	1·0	1·1	1·2	1·3	1·4	1·5	1·6
Y :	7·989	8·403	8·781	9·129	9·451	9·750	10·031

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1·1$ 5+5=10

(c) Define numerical integration.

Obtain a general quadrature formula for

$$\int_a^b f(x)\,dx.$$

Hence deduce Simpson's $\frac{1}{3}$rd rule.

1+5+4=10

OR

Write a short note on Euler's method. Give the geometric interpretation of Euler's method.

Give an algebraic interpretation of Euler's method.

Solve by using Euler's method :

$y' = x + y$; $y(0) = 2$ for $0 \le x \le 1$

$h = 0·5$ 2+2+2+4=10