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3 (Sem-6) MAT M 5

2020

MATHEMATICS

(Major)

Paper : 6.5

(Graph and Combinatorics)

Full Marks : 60

Time : Three hours

**The figures in the margin indicate
full marks for the questions.**

1. Answer the following questions :

1×7=7

(a) The value of $2P(n, n-2)$ is

(i) $P(2n, n)$

(ii) $P(n, n-2)$

(iii) $P(n, n)$

(iv) None of these.

Contd.

(b) Find how many functions are there from X to Y where $X = \{1, 2, 3\}$, $Y = \{a, b, c\}$.

(c) The number of vertices of odd degree in a graph is —

- (i) always even
- (ii) always odd
- (iii) can be even as well as odd
- (iv) None of above.

(d) The number of vertex in a loop is :

- (i) 0
- (ii) 1
- (iii) 2
- (iv) 4

(e) Which of the following statements are true ?

- (i) Every cycle is a Hamiltonian graph.
- (ii) Any graph obtained by adding edges to a Hamiltonian graph is also Hamiltonian.
- (iii) A Hamiltonian graph always has a pendent vertex.
- (iv) Trees are always Hamiltonian.

(f) Determine True **or** False of the following statement :

“ $K_{3,3}$ is non-planar”.

(g) Define Eulerian graph.

2. Answer the following questions : $2 \times 4 = 8$

(a) Prove that every graph is an intersection graph.

(b) Represent the graph $G(V, E)$ where the vertex set V and the edge set E are as follows :

$$V = \{1, 2, 3, 4\}$$

$$E = \{(x, y) : x + y \text{ is odd}\}$$

(c) A connected planar graph has nine vertices having degrees 2, 2, 2, 3, 3, 3, 4, 4 and 5. How many edges are there ?

(d) Does there exist a tree T with 8 vertices such that, the sum of degree of vertices is 16 ? Justify your answer.

3. Answer the following questions : $5 \times 3 = 15$

(a) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3 and 7.

(b) There exists no simple graph corresponding to the following degree sequences 2, 2, 4, 4, 2. Justify the above statement.

Or

Show that a complete graph with n

vertices consists of $\frac{n(n-1)}{2}$ edges.

(c) Prove that a connected graph is bipartite if and only if it contains no odd cycles.

Or

If a graph G is a tree then prove that every two vertices of G are joined by unique path.

4. Answer **any one** part :

(a) Prove that a connected graph G is Eulerian if and only if every vertex of G has even degree. 10

(b) (i) For a graph G , prove that

$$K(G) \leq \lambda(G) \leq \delta(G)$$

The symbols have their usual meaning. 6

(ii) Among all graphs with p vertices and q edges, prove that the maximum connectivity is 0 when

$$q < p-1 \text{ and } \left\lfloor \frac{2q}{p} \right\rfloor \text{ when } q \geq p-1.$$

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5. Answer **any one** part : 10

(a) If in a graph G has $n \geq 3$ vertices and every vertices has degree at least $\frac{n}{2}$,

then G is Hamiltonian.

(b) Let G be a graph of n vertices. If the sum of the degrees of each pair of vertices in G is $n-1$ or larger, then prove that there exists a Hamiltonian path in G .

6. Answer **any one** part :

(a) (i) In how many ways can 21 identical books on English and 19 identical books on Hindi be placed in a row on a shelf, so that two books on Hindi may not be together?

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(ii) Enumerate the number of non-negative integral to the inequality

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 19$$

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(b) (i) How many outcomes are possible by casting a 6 faced die 10 times?

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(ii) How many integers solution are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 13, \quad 0 \leq x_i \leq 5;$$

$i = 1, 2, 3, 4?$

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