3 (Sem–6/CBCS) MAT HC 2

2023

MATHEMATICS

(Honours Core)

Paper : MAT–HC–6026

(Partial Differential Equation)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following : 1×7=7

   (i) The first order, quasi linear and linear partial differential equation are solved by using

   (a) Lagrange's method

   (b) Charpit's method

(c) Jacobi method

(d) None of the above

(Choose the correct answer)

(ii) The partial differential equation

$$x\left(\frac{\partial^2 z}{\partial x^2}\right)+\frac{\partial^2 z}{\partial y^2}=x^2$$ is classified as

(a) Parabolic, $x=0$

(b) Elliptic, $x>0$

(c) Hyperbolic, $x<0$

(d) All of the above

(Choose the correct answer)

(iii) What are the order and degree of

$$\frac{\partial^2 z}{\partial x^2}=\sqrt{1+\frac{\partial z}{\partial y}}\ ?$$

(iv) What type of partial differential equation is readily solved by Charpit's method ?

(v) The equation $p^2+q^2=1$ is

(a) linear

(b) semi linear

(c) quasi linear

(d) Non-linear

(Choose the correct answer)

(vi) The solution which has number of arbitrary constants equal to number of independent variables is

(a) general integral

(b) complete integral

(c) particular integral

(d) singular integral

(Choose the correct answer)

(vii) Write down the form obtained of the PDE, in a function $X(x, y)$ and two variables x, y after seperation of variables is applied.

2. Answer in short : 2×4=8

(i) Write down the construction of a first order partial differential equation.

(ii) Define partial differential equation. Give *one* example.

(iii) Eliminate arbitrary constants from $z = Ae^{pt} \sin px$ to form a partial differential equation.

(iv) Determine whether the given equation is parabolic, elliptic or hyperbolic

$$y^2 \frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

3. Answer ***any three*** : 5×3=15

(i) Eliminate the arbitrary function f from the equation

$$f\left(x^2 + y^2 + z^2, z^2 - xy\right) = 0$$

(ii) Find the general integrals of the linear partial differential equations

$$z(xp - yq) = y^2 - x^2$$

(iii) Find the equation of the integral surface of the differential equation $2y(z-3)p + (2x-z)q = y(2x-3)$ which passes through the circle $z = 0$, $x^2 + y^2 = 2x$.

(iv) Reduce to canonical form and find the general solution of $u_x + u_y = u$.

(v) Apply the method of seperation of variables $u(x, y) = f(x)\, g(y)$ to solve the equation $y^2 u_x^2 + x^2 u_y^2 = (xyu)^2$.

4. Answer the following questions : 10×3=30

(a) Find a complete integral of $\left(p^2 + q^2\right) y = qz$ by Charpit's method.

Or

Apply the method of seperation of variables $u(x, y) = f(x)\, g(y)$ to solve the equation $u_x + 2u_y = 0$, $u(0, y) = 3e^{-2y}$.

(b) Solve $p_3 x_3 (p_1 + p_2) + x_1 + x_2 = 0$ by Jacobi method.

Or

Transform the equation to canonical form $u_{xx} + y^2 u_{yy} = y$.

(c) Obtain the general solution of the equation

$$x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} + xy\, u_x + y^2 u_y = 0$$

Or

Solve the following :

(i) $x\left(y^2 - z^2\right)p + y\left(z^2 - x^2\right)q = z\left(x^2 - y^2\right)$

(ii) $\left(x^2 - y^2 - z^2\right)p + 2xyq = 2xz$
