2023

MATHEMATICS

(Honours Core)

Paper : MAT–HC–2026

(Differential Equation)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

   (a) What is meant by implicit solution of a differential equation ?

   (b) Find the Wronskian of the set $\{e^x, e^{-x}\}$.

   (c) Determine whether the differential equation $2xy\,dx + (1+x^2)\,dy = 0$ is exact.

(d) Find the general solution of

$$y^{(3)} - 6y'' + 11y' - 6y = 2xe^x$$

(e) Consider the differential equation

$$(3y + 4xy^2)\,dx + (2x + 3x^2y)\,dy = 0$$

(i) Show that this equation is not exact.

(ii) Find the integrating factor of the differential equation and hence solve it.

1+4=5

4. Answer the following questions : ***(any three)*** 10×3=30

(a) Suppose that the functions $M(x, y)$ and $N(x, y)$ are continuous and have continuous first-order partial derivatives in the region R in xy-plane. Prove that the differential equation

$$M(x, y)\,dx + N(x, y)\,dy = 0$$

is exact if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

at each point of R.

Or

Solve the differential equation by making suitable transformation

$$(x - 2y + 1)\,dx + (4x - 3y - 6)\,dy = 0$$

(b) In a fish farm, fish are harvested at a constant rate of 2100 fish per week. The per capita death rate for the fish is 0·2 fish per day per fish and the per capita birth rate is 0·7 fish per day per fish.

(i) Write a word equation describing the rate of change of the fish population. Hence obtain a differential equation for the population of fish.

(ii) If the fish population at a given term is 2,40,000; give an estimate of the number of fish born in one week.

(iii) Determine if there are any value for which the fish population is in equilibrium.

2+3+5=10

Or

(i) A culture initially has N_0 number of bacteria. At $t = 1$ hour the number of bacteria is measured to be $\frac{3}{2}N_0$. If the rate of growth is proportional to the number of bacteria present, determine the time necessary for the number of bacteria to triple. 5

(ii) The differential equation $\frac{dC}{dt} = \frac{F}{V}(C_{in} - C)$ describes the level of pollution in a lake, where V is the volume of the lake, F is the flow, C is the concentration of pollution at time t and C_{in} is the concentration of pollution entering the lake.

(i) Solve the differential equation with the initial condition $C(0) = C_0$.

(ii) How long will it take for the lake's pollution level to reach 5% of its initial level if only fresh water flows into the lake?

3+2=5

(c) (i) Find the general solution of

$$y^{(3)} + y'' = 3e^x + 4x^2$$

(ii) Solve the Euler equation

$$x^2y'' + xy' + 9y = 0$$

5+5=10

Or

Solve the initial value problem

$$y'' - 3y' + 2y = 3e^{-x} - 10\cos 3x$$

given $y(0) = 1, y'(0) = 2$. 10
