# Total number of printed pages-31

## 3 (Sem-3/CBCS) MAT HG 1/RC/HG 2

#### 2022

#### **MATHEMATICS**

(Honours Generic/Regular)

Answer the Questions from any one Option.

### OPTION-A

Paper: MAT-HG-3016 / MAT-RC-3016

(Differential Equations)

## **OPTION-B**

Paper: MAT-HG-3026

(Linear Programming)

Full Marks: 80

Time: Three hours

The figures in the margin indicate full marks for the questions.

#### **OPTION-A**

Paper: MAT-HG-3016 /MAT-RC-3016

(Differential Equations)

Answer either in English or in Assamese.

1. Answer the following questions: (any ten)

1×10=10

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ কৰাঃ (যিকোনো দহটা)

(a) Write down the order of the following differential equation:

তলৰ অৱকল সমীকৰণটোৰ ক্ৰম লিখা ঃ

$$\frac{d^6x}{dt^6} + \left(\frac{d^4x}{dt^4}\right) \left(\frac{d^3x}{dt^3}\right) + x = t$$

(b) What is meant by implicit solution of a differential equation?

এটা অৱকল সমীকৰণৰ অন্তৰ্নিহিত সমাধান মানে কি ?

(c) Form the differential equation of the family of circles  $x^2 + y^2 = a^2$ .

 $x^2+y^2=a^2$  বৃত্তৰ পৰিয়ালটোৰ অৱকল সমীকৰণটো গঠন কৰা।

- (d) Define an exact differential equation.
  এটা যথাৰ্থ অৱকল সমীকৰণৰ সংজ্ঞা লিখা।
- (e) Evaluate the Wronskian of the functions sinx and cosx.
  sinx আৰু cosx ফলন দুটাৰ Wronskian নির্ণয়
  কৰা।

(g) Check exactness of (যথাৰ্থতা পৰীক্ষা কৰা) ঃ 
$$(x^2 + 2y^2) dx + (4xy - y^2) dy = 0$$

(i) Write the UC set corresponding to the UC function 
$$x^2e^x$$
.

UC ফলন  $x^2e^x$  সাপেক্ষে UC সংহতিটো লিখা।

(j) If  $e^{2x}$  and  $e^{3x}$  are two linearly independent solutions of a 2nd order linear differential equation, write down the general solution.

 $e^{2x}$  আৰু  $e^{3x}$  এটা দ্বিমাত্ৰাৰ অৱকল সমীকৰণৰ দুটা ৰৈখিকভাৱে স্বতন্ত্ৰ সমাধান হ'লে সমীকৰণটোৰ সাধাৰণ সমাধান লিখা। (k) The roots of the auxiliary equation corresponding to a 5th order linear differential equation are 2, 2, 3 ± 4i. Write the general solution of the equation.

এটা 5 মাত্ৰাৰ ৰৈখিক অৱকল সমীকৰণৰ সহায়ক সমীকৰণটোৰ মূল কেইটা 2, 2, 3 ± 4i হ'লে সমীকৰণটোৰ সাধাৰণ সমাধান লিখা।

(1) Consider the equation

$$(2x-5y) dx + (4x-y) dy = 0$$

What transformation will reduce it to a separable equation?

$$(2x-5y) dx + (4x-y) dy = 0$$
 সমীকৰণটোক কি ৰূপান্তৰে এটা বিয়োজিত (separable) সমীকৰণলৈ সমানীত কৰিব ?

(m) Determine the integrating factor of:
অনুকলন গুণক উলিওৱাঃ

$$\frac{dy}{dx} + \frac{3y}{x} = 6x^2$$

(n) In the differential equation

$$M(x, y)dx + N(x, y)dy = 0$$
, if

$$\frac{1}{N(x,y)} \left[ \frac{\partial M(x,y)}{\partial y} - \frac{\partial N(x,y)}{\partial x} \right] \text{ depends}$$

upon x only, what will be the integrating factor of the equation?

$$M(x, y)dx + N(x, y)dy = 0$$

অৱকল সমীকৰণটোৰ যদিহে

$$\frac{1}{N(x,y)} \left[ \frac{\partial M(x,y)}{\partial y} - \frac{\partial N(x,y)}{\partial x} \right]$$

অকল x ৰ নিৰ্ভৰশীল হয় তেন্তে সমীকৰণটোৰ অনুকলন গুণক কি ?

(o) Solve (স্মাধান কৰা):

$$ydx + xdy = 0$$

(p) Write down the general form of Cauchy-Euler equation of order n.
n মাত্ৰাৰ কচি-ইউলাৰ সমীকৰণৰ সাধাৰণ ৰূপটো লিখা। (q) Is the equation linear?
সমীকৰণটো ৰৈখিক হয়নে ?

$$\frac{d^2y}{dx^2} + y\frac{dy}{dx} + x = 0$$

(r) Write down the UC set corresponding to UC function sinx.

UC ফলন sin x সাপেক্ষে UC সংহতিটো লিখা।

- Answer the following questions: (any five)
   2×5=10
   তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ কৰা ঃ (যিকোনো পাঁচটা)
  - (a) Determine all values of constant m for which  $y = e^{mx}$  is a solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0.$$

m ब সকলো মান নির্ণয় কৰা, যাৰ বাবে

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$$
 সমীকৰণটোৰ  $y = e^{mx}$   
এটা সমাধান হয়।

(b) What is meant by singular solution of a differential equation?

এটা অৱকল সমীকৰণৰ একক সমাধান বুলিলে কি বুজা ?

(c) Write down the complementary function of the differential equation

$$\frac{d^2y}{dx^2} - y = \tan x.$$

$$\frac{d^2y}{dx^2} - y = \tan x$$
 অৱকল সমীকৰণটোৰ পৰিপূৰক ফলনটো লিখা।

(d) Determine the most general function M(x, y) such that the equation

$$M(x, y)dx + (2x^2y^3 + x^4y)dy = 0$$
is exact.

অত্যন্ত সাধাৰণ ফলন M(x,y) উলিওৱা যাতে,

$$M(x, y)dx + (2x^2y^3 + x^4y)dy = 0$$

সমীকৰণটো যথাৰ্থ হয়।

(e) Show that the differential equation  $(x^2 - 3y^2)dx + 2xy dy = 0$  is homogeneous.

দেখুওৱা যে  $(x^2 - 3y^2)dx + 2xydy = 0$  অৱকল সমীকৰণটো সমমাত্ৰিক।

(f) Show that the ordered pair of functions  $(3e^{7t}, 2e^{7t})$  is a solution of the linear system:

দেখুওৱা যে ক্ৰমিত যুগ্ম ফলন  $\left(3e^{7t}, 2e^{7t}\right)$  তলৰ ৰৈখিক প্ৰণালীটোৰ এটা সমাধান হয় ঃ

$$\frac{dx}{dt} = 5x + 3y$$

$$\frac{dy}{dt} = 4x + y$$

(g) Write down the form of particular solution for the differential equation:

তলৰ অৱকল সমীকৰণটোৰ বিশেষ সমাধান (particular solution) ৰ ৰূপটো লিখাঃ

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4x^2$$

(h) Solve (সমাধান কৰা): 
$$x \sin y dx + (x^2 + 1)\cos y dy = 0$$

(i) Reduce the Bernoulli's equation 
$$x\frac{dy}{dx}+y=-2x^6y^4 \text{ to linear equation by appropriate transformation.}$$
 উপযুক্ত ৰূপান্তৰ সহায়ত বাৰ্নোলীৰ সমীকৰণ 
$$x\frac{dy}{dx}+y=-2x^6y^4 \text{ ক ৰৈখিক সমীকৰণলৈ}$$

(j) Find the general solution : সাধাৰণ সমাধান উলিওৱা :

সমানীত কৰা।

$$4\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 5y = 0$$

3. Answer the following questions : (any four)
5×4=20
তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ কৰা ঃ (যিকোনো চাৰটা)

(a) Show that the relation  $x^2 + y^2 - 25 = 0$  is an implicit solution of the differential equation  $x + y \frac{dy}{dx} = 0$  on the interval I defined by -5 < x < 5.

দেখুওৱা যে -5 < x < 5 অন্তৰালত

$$x + y \frac{dy}{dx} = 0$$
 অৱকল সমীকৰণটোৰ

$$x^2 + y^2 - 25 = 0$$
 এটা অন্তর্নিহিত সমাধান হয়।

- (b) Write down the general form of a Bernoulli equation. Describe the method of reducing this equation to a linear equation. 1+4=5
  বার্নোলী সমীকৰণৰ সাধাৰণ ৰূপটো লিখা। এই সমীকৰণক এটা ৰৈখিক সমীকৰণলৈ সমানীত কৰা পদ্ধতিটো ব্যাখ্যা কৰা।
- (c) Solve (সমাধান কৰা): (3x y 6)dx + (x + y + 2)dy = 0
- (d) Reduce to first order differential equation and then solve: 1+4=5
  এক মাত্ৰাৰ (ক্ৰমৰ) অৱকল সমীকৰণলৈ সমানীত কৰি

$$y'' + y' = 0$$

সমাধান কৰা ঃ

(e) Solve the Cauchy-Euler equation : কচি-ইউলাৰ সমীকৰণটো সমাধান কৰা ঃ

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^3$$

(f) Determine the constant A such that the following equation is exact:

Aৰ মান উলিওৱা যাতে তলৰ অৱকল সমীকৰণটো যথাৰ্থ হয় ঃ

$$(Ax^2y + 2y^2)dx + (x^3 + 4xy)dy = 0$$

Hence solve the resulting exact equation. 2+3=5

লগতে লব্ধ যথাৰ্থ সমীকৰণটো সমাধান কৰা।

(g) Show that x=t+1, y=-5t-2 is a particular solution of the non-homogeneous linear system

$$\frac{dx}{dt} = 5x + 2y + 5t$$

$$\frac{dy}{dt} = 3x + 4y + 17t$$

Write the general solution of the system.

দেখুওৱা যে x = t + 1, y = -5t - 2 তলৰ অসমমাত্রিক ৰৈখিক প্রণালীটোৰ সাধাৰণ সমাধান হয়  $\epsilon$ 

$$\frac{dx}{dt} = 5x + 2y + 5t$$
$$\frac{dy}{dt} = 3x + 4y + 17t$$

লগতে প্ৰণালীটোৰ সাধাৰণ সমাধান লিখা।

(h) Solve the initial value problem : আদিমান বিশিষ্ট সমীকৰণটো সমাধান কৰা ঃ

$$\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 10y = 0, \ y(0) = -4,$$
  
y'(0) = 2

4. Answer the following questions: (any four) 10×4=40

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ কৰাঃ (যিকোনো চাৰটা)

(a) Prove that the linear differential equation  $\frac{dy}{dx} + P(x)y = Q(x)$  has an integrating factor of the form  $e^{\int P(x)dx}$  and one-parameter family of solution

$$y.e^{\int P(x)dx} = \int e^{\int P(x)dx} Q(x)dx + C$$
7+3=10

প্ৰমাণ কৰা যে ৰৈখিক অৱকল সমীকৰণ

$$\frac{dy}{dx}$$
 +  $P(x)y = Q(x)$  ৰ  $e^{\int P(x)dx}$  এটা অনুকলন গুণক হয় আৰু সমীকৰণটোৰ এক চলকযুক্ত সমাধান হ'ল

$$y.e^{\int P(x)dx} = \int e^{\int P(x)dx}Q(x)dx + C$$

(b) (i) Find the orthogonal trajectories of the family of parabolas  $y = cx^2$ .

 $y = cx^2$  অধিবৃত্তৰ পৰিয়ালটোৰ লাম্বিক প্ৰক্ষেপপথ নিৰ্ণয় কৰা।

(ii) Find a family of oblique trajectories that intersect the family of circles  $x^2 + y^2 = c^2$  at an angle 45°.

 $x^2 + y^2 = c^2$  বৃত্তৰ পৰিয়ালটোক  $45^\circ$  কোণত ছেদ কৰি থকা এটা তিৰ্যক প্ৰক্ষেপপথৰ পৰিয়াল উলিওৱা।

$$\frac{dy}{dx} + y = f(x)$$
 where

$$f(x) = \begin{cases} 5, & 0 \le x < 10 \\ 1, & x \ge 10 \end{cases} \text{ and } y(0) = 6$$

আদিমান বিশিষ্ট সমীকৰণ

$$\frac{dy}{dx} + y = f(x)$$
 সমাধান কৰা য'ত

$$f(x) = \begin{cases} 5, & 0 \le x < 10 \\ 1, & x \ge 10 \end{cases}$$

আৰু y(0) = 6.

(d) Solve by method of variation of parameter:

প্ৰাচল বিচৰণ পদ্ধতিৰে সমাধান কৰাঃ

$$\frac{d^2y}{dx^2} + y = \tan x \sec x$$

<sup>(</sup>c) Solve the initial value problem

(e) Consider the differential equation অৱকল সমীকৰণ এটা লোৱা হ'ল

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

- i) Show that  $e^x$  and  $xe^x$  are linearly independent solutions of this equation on the interval  $-\infty < x < \infty$ . 5
  দেখুওৱা যে  $-\infty < x < \infty$  অন্তৰালত  $e^x$  আৰু  $xe^x$  সমীকৰণটোৰ দুটা ৰৈখিকভাৱে স্বতন্ত্ৰ সমাধান হয়।
- (ii) Write the general solution of the equation. 2
  সমীকৰণটোৰ সাধাৰণ সমাধান লিখা।
- (iii) Find the solution that satisfies the condition y(0)=1, y'(0)=4.

  Explain why this solution is unique. 2+1=3 y(0)=1, y'(0)=4 চর্ত্ত সাপেকে

  সমীকৰণটোৰ সমাধান উলিওৱা।

  এই সমাধান কিয় একক হয়, ব্যাখ্যা কৰা।

(f) Find the general solution by the method of undetermined co-efficients:

অনিৰ্ধাৰিত সহগ পদ্ধতিৰে সাধাৰণ সমাধান উলিওৱা ঃ

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 6\sin 2x + 7\cos 2x$$

(g) Consider the linear system ৰৈখিক প্ৰণালী এটা লোৱা হ'ল

$$\frac{dx}{dt} = 5x + 2y$$

$$\frac{dy}{dt} = 3x + 4y$$

(i) Show that (দেখুওৱা যে)

$$x = 2e^{2t}$$
,  $x = e^{7t}$ 

and (আৰু)

$$y = 3e^{2t}$$
,  $y = e^{7t}$ 

are solutions of this system. এই প্ৰণালীটোৰ সমাধান হয়।

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(ii) Show that the two solutions defined in part (i) are linearly independent on every interval 
$$a \le t \le b$$
.

দেখুওৱা যে part (i) ত উল্লিখিত সমাধান দুটা  $a \le t \le b$  অন্তৰালত ৰৈখিকভাৱে স্বতন্ত্ৰ হয়।

(iii) Write the general solution of the system.

প্রণালীটোৰ সাধাৰণ সমাধান লিখা।

- (h) Solve the following: (সমাধান কৰা) 5+5=10
- (i)  $2x(y+1)dx (x^2+1)dy = 0$ , y(1) = -5
- (ii)  $(2x \sin y + y^3 e^x) dx + (x^2 \cos y + 3y^2 e^x) dy = 0$
- (i) (i) Given that y = x+1 is a solution of  $(x+1)^2 \frac{d^2y}{dx^2} 3(x+1)\frac{dy}{dx} + 3y = 0$ .

Find a linearly independent solution by reducing the order.

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 3y = 0$$

অৱকল সমীকৰণটোৰ এটা সমাধান y = x + 1 হয়। সমীকৰণটোৰ ক্ৰম লঘুকৃত কৰি এটা ৰৈখিকভাৱে স্বতন্ত্ৰ সমাধান উলিওৱা।

(ii) Given that  $e^{-x}$ ,  $e^{3x}$  and  $e^{4x}$  are all solutions of

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0$$

Show that they are linearly independent on the interval  $-\infty < x < \infty$ .

দিয়া আছে যে  $e^{-x}$ ,  $e^{3x}$  আৰু  $e^{4x}$  আটাইবোৰেই

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0$$

অৱকল সমীকৰণটোৰ সমাধান হয়।

দেখুওৱা যে  $-\infty < x < \infty$  অন্তৰালত সমাধানবোৰ ৰৈখিকভাৱে স্বতন্ত্ৰ।

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Contd.

(j) Find the general solution: 5+5=10

সাধাৰণ সমাধান উলিওৱা ঃ

(i) 
$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 3x^2$$

(ii) 
$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$$

#### OPTION-B

Paper: MAT-HG-3026 (Linear Programming)

1. Answer any ten of the following:

1×10=10

- (i) Define feasible solution of a linear programming problem (LPP).
- (ii) If a given LPP has two feasible solutions, then how many feasible solutions are there for the LPP?
- (iii) How many basic solutions are possible in a system of m simultaneous linear equations in n > m unknowns?
- (iv) When is a basic solution to the system of equations Ax = b said to be degenerate?
- (v) Define surplus variable.
- (vi) When is an LPP said to be in standard format?
- (vii) Define hyperplane.
- (viii) "All boundary points of a convex set are necessarily extreme points." Is it true?

Maximize 
$$3x_1 - 2x_2$$

subject to 
$$x_1 + x_2 \le 1$$
  
 $2x_1 + 2x_2 \ge 4$   
 $x_1, x_2 \ge 0$ 

have an optimal solution?

- (x) Name two methods that can be employed to solve LPP having artificial variables.
- (xi) Consider the primal problem given as Minimize  $x_1 3x_2 2x_3$

subject to 
$$3x_1 - x_2 + 2x_3 \le 7$$
  
 $2x_1 - 4x_2 \ge 12$   
 $-4x_1 + 3x_2 + 4x_3 = 10$ 

 $x_1, x_2 \ge 0$  and  $x_3$  unrestricted.

Can the dual of this primal have unrestricted variables?

(xii) Write the relation between  $Z_P$  and  $Z_D$ , where  $Z_P$  is the optimal value of the primal objective function and  $Z_D$  is the optimal value of the dual objective function.

- (xiii) A primal problem has 7 constraints in 3 variables. How many constraints are there in its dual?
- (xiv) When is a transportation problem said to be unbalanced?
- (xv) Write the full form of VAM.
- (xvi) What is a fair game?
- (xvii) Is it necessary that a game should always pass a saddle point?
- (xviii) Can a two-person zero-sum game in normal form be solved as an LPP?
- 2. Answer any five of the following: 2×5=10
  - (i) Define basic feasible solution (B.F.S.) of an LPP. When is a B.F.S. said to be non-degenerate?
  - (ii) Explain the following terms in the context of LPP:
    - (a) Objective function
    - (b) Decision variables
  - (iii) Show that a hyperplane is a convex set.

(iv) Solve the following LPP graphically:

Maximize 
$$Z = 4x_1 + 4x_2$$
  
subject to  $x_1 + x_2 \le 5$   
 $3x_1 + x_2 \le 9$ 

$$x_1, x_2 \ge 0$$

- (v) What is meant by unbounded solution in linear programming?
- (vi) Write the dual of the following primal problem:

$$Minimize Z_P = 15x_1 + 12x_2$$

subject to 
$$x_1 + 2x_2 \ge 3$$
  
 $2x_1 - 4x_2 \le 5$ 

$$x_1, x_2 \ge 0$$

- (vii) State the fundamental theorem of duality.
- (viii) Find an initial basic feasible solution to the following transportation problem by least cost method:

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$O_{\mathrm{I}}$	2	1	3	4	30
$O_2$	3	2	1	4	. 50
$O_3$	5	2	3	8	20
Demand	20	40	30	10	

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- (ix) State the mathematical formulation of an assignment problem.
- (x) In a two-person zero-sum game, the pay-off matrix is given by

Find its saddle points.

- 3. Answer any four of the following: 5×4=20
  - (i) Define convex set and show that the intersection of any finite number of convex sets is a convex set.
  - (ii) Show that every basic feasible solution of an LPP is an extreme point of the convex set of its feasible solutions.
  - (iii) Solve the following LPP by simplex method:

Maximize 
$$Z = 3x_1 + 2x_2$$

subject to 
$$x_1 + x_2 \le 4$$
  
 $x_1 - x_2 \le 2$   
 $x_1, x_2 \ge 0$ 

$$Maximize Z = 2x_1 + 3x_2$$

subject to 
$$x_1 + 2x_2 \le 4$$
  
 $x_1 + x_2 = 3$   
 $x_1, x_2 \ge 0$ 

(v) Find the dual of the following primal problem:

Maximize 
$$2x_1 + x_2$$

subject to 
$$x_1 + 5x_2 \le 10$$
  
 $x_1 + 3x_2 \ge 6$   
 $x_1 + x_2 \le 4$ 

 $x_2 \ge 0$  and  $x_1$  unrestricted

(vi) Use north-west corner method to find an initial basic feasible solution to the following transportation problem:

·	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	Supply
$O_1$	2	11	10	3	7	4
$O_2$	1	4	7	2	1	8
O <sub>3</sub>	3	9	4	8	12	9
Demand	3	3	4	5	6	J

# (vii) Find an optimal solution to the following assignment problem:

	1.	II	III	IV_
A	.12	30	21	15
<b>B</b> .	18	33	9	31
С	44	25	24	21
Ď	23	30	28	14

(viii) The pay-off matrix of a two-person zerosum game is given below:

				В		
,		Ι	II	III	IV	V
	I	9	3	1	8	0
Α	II.	6	5	4	6	7
	III	2	4	3	3	8
	IV	5	6	2	2	1

Find the best strategy for each player and the value of the game.

10×4=40

(i) Show that the following system of linear equations has a degenerate solution:

$$2x_1 + x_2 - x_3 = 2$$
$$3x_1 + 2x_2 + x_3 = 3$$

(ii) Reduce the feasible solution

$$x_1 = 2$$
,  $x_2 = 3$ ,  $x_3 = 1$   
of the following system of linear equations to a basic feasible solution:

$$2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14$$

(iii) Explain the simplex procedure to solve a linear programming problem (LPP).

(iv) Use two-phase method to solve the LPP:

Maximize 
$$Z = 5x_1 - 4x_2 + 3x_3$$

subject to 
$$2x_1 + x_2 - 6x_3 = 20$$
$$6x_1 + 5x_2 + 10x_3 \le 76$$
$$8x_1 - 3x_2 + 6x_3 \le 50$$
$$x_1, x_2, x_3 \ge 0$$

(v) Use Big-M method to solve the LPP:

Minimize 
$$Z = 4x_1 + x_2$$

subject to 
$$3x_1 + x_2 = 3$$
  
 $4x_1 + 3x_2 \ge 6$   
 $x_1 + 2x_2 \le 4$   
 $x_1, x_2 \ge 0$ 

(vii) (a) Write the dual of the LPP: 5

Minimize 
$$x_1 + x_2 + x_3$$

subject to 
$$x_1 - 3x_2 + 4x_3 = 5$$
  
 $2x_1 - 2x_2 \le 3$   
 $2x_2 - x_3 \ge 5$ 

 $x_1, x_2 \ge 0$  and  $x_3$  unrestricted.

Maximize 
$$3x_1 - 2x_2$$

subject to 
$$x_1 \le 4$$
  
 $x_2 \le 6$   
 $x_1 + x_2 \le 5$   
 $x_2 \ge 1$   
 $x_1, x_2 \ge 0$ 

(viii) Find and optimal solution to the following transportation problem:

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$W_1$	19	14	23	11.	11
$W_2$	15	16	12	21	13
<i>W</i> <sub>3</sub>	30	25	16	39	19
Demand	6	10	12	15	1

(ix) Apply the Hungarian method to solve the following assignment problem:

	I	II .	II III	
$A_{\cdot}$	12	10	.8	9
<b>B</b>	8	9	11	7
C	· 11	14	12	10
D	9	9	8	9

- (x) (a) What is game theory?
  - (b) Describe a two-person zero-sum game. Also mention any two basic assumptions in it.
  - (c) Explain the following terms: 2+2=4

Pure strategy, Mixed strategy