Total number of printed pages–8

3 (Sem–3/CBCS) PHY HC 1

2022

PHYSICS

(Honours)

Paper : PHY–HC–3016

(Mathematical Physics–II)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** of the following questions : 1×7=7

   (a) Define the singular point of a second order linear differential equation.

   (b) If $P_n(x)$ and $Q_n(x)$ are two independent solutions of Legendre equation, then write the general solution of the Legendre equation.

   (c) Give *one* example where Hermite polynomial is used in physics.

Contd.

(d) The function $P_n(1)$ is given as

(i) zero

(ii) -1

(iii) $P_n(-1)$

(iv) 1

(Choose the correct option)

(e) Define trace of a matrix.

(f) What is the rank of a zero matrix ?

(g) Define self-adjoint matrix.

(h) What do you mean by eigenvector ?

(i) Which one of the following represents an equation of a vibrating string ?

(i) $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

(ii) $\frac{\partial y}{\partial t} = c \frac{\partial y}{\partial x}$

(iii) None of the above

(Choose the correct option)

(j) Write the Laplace equation spherical polar co-ordinate system.

(k) Define gamma function.

(l) State the Dirichlet condition for Fourier series.

2. Answer ***any four*** of the following questions : 2×4=8

(a) Check whether Frobenius method can be applied or not to the following equation :

$$2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$$

(b) If $\int_{-1}^{+1} P_n(x)dx = 2,$ find the value of n.

(c) If A and B are Hermitian matrices, show that $AB+BA$ is Hermitian whereas $AB-BA$ is skew-Hermitian.

(d) Verify that $(AB)^T = B^T A^T$, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$$

(e) Given matrices

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

show that $\sigma_1\sigma_2 - \sigma_2\sigma_1 = 2i\sigma_3$.

(f) Using the property of gamma function evaluate the integral

$$\int_0^\infty x^4 e^{-x} dx$$

Contd.

(g) Write the degree and order of the following partial differential equations :

(i) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$

(ii) $\left(\frac{\partial u}{\partial x}\right)^3+\frac{\partial u}{\partial t}=0$

(h) Find the value of a_0 of the Fourier series for the function $f(x)=x\cos x$ in the interval $-\pi < x < \pi$.

3. Answer ***any three*** of the following questions : 5×3=15

(a) (i) Why is the function $\left(1-2xh+h^2\right)^{-1/2}$ known as a generating function of Legendre polynomial ? 1

(ii) Show that

$$\left(1-2xh+h^2\right)^{-1/2}=\sum_{n=0}^{\infty}P_n(x)h^n$$

where $P_n(x)$ is the Legendre polynomial. 4

(b) Evaluate explicitly the Legendre's polynomials $P_2(x)$ and $P_3(x)$. 2½+2½=5

(c) Write the recursion formula for gamma function. Prove that

$$\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}=1{\cdot}772$$

(d) What is diagonalize matrix ? Diagonalize the following matrix : 1+4=5

$$A=\begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}$$

(e) Express the matrix :

$A=\begin{bmatrix}2 & 1 & 0\\ 1 & -1 & -2\\ 4 & 2 & 0\end{bmatrix}$ as a sum of symmetric and skew-symmetric matrix.

(f) What is adjoint of a matrix ? For the matrix $A=\begin{bmatrix}1 & 2\\ 3 & -5\end{bmatrix}$ verify the theorem

$A\cdot(Adj A)=(Adj A)\cdot A=|A|\cdot I$

where I is unit matrix. 1+4=5

(g) If the solution $y(x)$ of Hermite's differential equation is written as

$$y(x)=\sum_{r=0}^{\infty}a_r x^{k+r}$$, show that the allowed values of k are zero and one only.

(h) Find the Fourier series representing

$$f(x)=x,\ 0<x<2\pi$$

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) Verify that the matrix

$$A=\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$$ is orthogonal. 2

(ii) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and also find A^{-1}. 5+3=8

(b) Obtain the power series solution of the Legendre equation

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$$

(c) (i) Obtain the following orthogonality property of Legendre polynomial :

$$\int_{-1}^{+1}P_n(x)P_m(x)dx=0 \text{ for } m\neq n$$ 6

(ii) Show that

$H_0(x)=1$ and $H_1(x)=2x$ 2+2=4

(d) Prove the following recurrence relations : 4+3+3=10

(i) $nP_n=(2n-1)xP_{n-1}-(n-1)P_{n-2}$

(ii) $xP_n'-P_{n-1}'=nP_n$

(iii) $2xH_n(x)=2nH_{n-1}(x)+H_{n+1}(x)$

(e) What is periodic function ? Express the periodic functions in a series of sine and cosine functions. What are Fourier coefficients ? Determine the Fourier coefficients. 1+1+1+7=10

(f) (i) Using the method of separation of variables, solve : 6

$$\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u, \text{ where } u(x,0)=6e^{-3x}$$

(ii) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ 4

(g) (i) If $H_n(x)$ be the polynomial of Hermite differential equation, prove that

$$\int_{-\infty}^{+\infty} e^{-x^2} H_n^2(x)\, dx = 2^n \sqrt{\pi}.n!$$ 7

(ii) Prove that the following matrix is unitary :

$$\begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{bmatrix}$$ 3

(h) Deduce the one dimensional wave equation of transversely vibrating string under tension T. Solve the equation by the method of separation of variables. 7+3=10