2019

MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) Write the nth derivative of $\frac{1}{ax+b}$.

(b) Write the value of $\frac{\partial f}{\partial y}$ for the function

$$f = ye^{-x/y}$$

(c) For a pedal curve $p = r \sin \phi$, write the formula of radius of curvature.

(d) Write the definition of cusp.

(e) Write the value of subnormal to the curve $y^2 = 4ax$ at the point (x, y).

(f) What is the value of $\int_0^1 xe^x dx$?

(g) A curve $y = f(x)$ rotates about x-axis to form a solid. Write the formula to find the volume of the solid bounded by $x = x_1$, $x = x_2$.

(h) For a curve $y = f(x)$, write the formula to find the length of the tangent.

(i) What is asymptote?

(j) Write the value of $\int_0^{\pi/2} \sin^7 \theta d\theta$.

2. Solve the following questions : 2×5=10

(a) If $y = a\cos(\log x) + b\sin(\log x)$, show that

$$x^2 y_2 + x y_1 + y = 0$$

(b) Show that the pedal equation of the curve $r = e^{\theta}$ is $2p^2 = r^2$.

(c) Find the area of the region bounded by the parabola $x^2 = 16y$ and its latus rectum.

(d) Find the area of a loop of the curve $r = a\cos 2\theta$.

(e) Evaluate :

$$\int_0^{\pi/2} \log \tan x \, dx$$

3. Answer the following questions : 5×2=10

(a) If $y = [x + \sqrt{1+x^2}]^m$, show that

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$$

(b) Find the asymptotes of the curve

$$x^4 - x^2y^2 + x^2 + y^2 - a^2 = 0$$

4. Answer any *one* part : 10

(a) (i) Find the equations of tangent and normal at the point t of the curve

$$x = a(t + \sin t),\ y = a(1 - \cos t)$$

(ii) Show that for the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

the radius of curvature at an extremity of the major axis is equal to half the latus rectum.

(b) (i) If $u = \log(x^2 + y^2 + z^2)$, show that

$$x\frac{\partial^2 u}{\partial y \partial z} = y\frac{\partial^2 u}{\partial z \partial x} = z\frac{\partial^2 u}{\partial x \partial y}$$

(ii) Evaluate :

$$\int \sqrt{10 - 4x + 4x^2}\ dx$$

5. Answer the following questions : 5×2=10

(a) If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

(b) Evaluate :

$$\int (x+2)\sqrt{2x^2+2x+1}\, dx$$

6. Answer any *one* part : 10

(a) *(i)* If $u = f(r)$, where $r = x^2 + y^2$, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

(ii) Trace the curve $y = x^3$.

(b) *(i)* Show that

$$\int_0^{\pi/2} \log\cos x dx = \frac{\pi}{2}\log\frac{1}{2}$$

(ii) Obtain a reduction formula for $\int \sec^n x dx$.

7. Answer any *two* parts : 5×2=10

(a) Obtain a reduction formula for

$$\int_0^1 x^m (1-x)^n \, dx$$

(b) Find the area of the region bounded that is inside the circle $r = \sin\theta$ and outside the cardioid $r = 1 - \cos\theta$.

(c) Find the length of the arc of the parabola $x^2 = 4by$ cut off by the line $x = 2y$.

8. Answer the following questions : 5+5=10

(a) Evaluate :

$$\int_0^1 \frac{dx}{(1+x)\sqrt{1+2x-x^2}}$$

Or

Show that points of inflexion of the curve $y^2 = (x-a)^2(x-b)$ lie on the line $3x + a = 4b$.

(b) Find the area above the x-axis included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$.

Or

Trace the curve $ay^2 = x^3$.

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